Study Guide for Benchmark #1
Mt 210 Calculus I
Sr. Barbara Reynolds
Window-of-Opportunity: 2 weeks at mid-semester (ending at mid-term)

Benchmark testing is the department's way of assuring that students have achieved minimum levels of computational skill. While partial credit is given on the other tests in this course, on the benchmark tests you are expected to demonstrate that you can do basic computations carefully and accurately. This benchmark covers both skills that are considered prerequisite for this course, and computations that have been covered in the first two chapters of this course.

This benchmark will be given during the last 30 minutes of class on Monday, (Week 7, Day 18). If you do not pass on your first attempt, you may retake this benchmark once or twice, up through (Week 8, Day 22).

This test will have ten basic computational problems. *You may not use a calculator while you are working on the benchmark.* To pass the benchmark, you must get 9 or 10 of the problems completely correct; there are no partial credits on the benchmark. If you pass the benchmark on your first attempt, I will record your grade as 100%. If you do not pass on your first attempt, you may meet with me to go over your mistakes and demonstrate that you have done some additional study. You may retake the benchmark once or twice up through DATE. However, you may not retake the benchmark twice in the same day. If you pass on a retake, your grade will be recorded as the average of your scores on each attempt. If you do not pass the benchmark in three attempts or by DATE, your score will be recorded as the lower of 40% or your average on three attempts. In calculating your average in this case, if you have not attempted to retake the Benchmark, your scores for these non-attempts will be considered as 0. So the idea is to do as well as you can on your first attempt, and to make sure that you do pass by DATE.

Your grade on this benchmark will contribute 5% to your overall grade for this course. In fact, passing this Benchmark on your first attempt (and earning that 100%) will bring up your current grade in this course. Failing to pass this Benchmark will bring your course grade down by at least half a letter grade.

This benchmark will have two problems from *basic algebra*, two from *coordinate geometry*, two from *trigonometry*, and four problems from *symbolic computation of the derivative*.

Basic Algebra *(2 problems)*
You should be able to demonstrate the following skills:

- Simplify an arithmetic or algebraic expression using correct order of operations and rules for working with exponents.
- Evaluate a given function, $f(x)$, at particular values, such as $x = 5$, $x = x + h$, or $x = g(x)$.
- Use rules of exponents to evaluate or simplify an algebraic equation.

- *Remember: A negative exponent denotes a reciprocal, and a fractional exponent denotes a root.*
- Calculate the sum or product of polynomials.
- Solve a linear equation for a specified value of *x* or *y*.
- Given an equation of the form $y = f(x) = Ax^n$ or $y = f(x) = Ae^{bx}$, you should be able to take logs of both sides of the equation, and simplify the resulting expression.

Coordinate Geometry *(2 problems)*
You should be able to demonstrate the following skills:
- Plot points in the coordinate plane using rectangular coordinates.
- Find the slope of a straight line given its graph, its algebraic expression, or a set of point which lie on the line.
- Use the slopes to determine whether two lines are parallel, perpendicular, or neither.
  - *Remember: Parallel lines have the same slope. If two lines are perpendicular, the product of their slopes will be -1.*
- Find the equation of a line given two points, or a point and the slope of the line.
- Find the length and the midpoint of a line segment.
- Sketch the graph of a line given its equation, its slope and one point, or two points.

- You should know and be able to use formulas for *area* of a square, rectangle, triangle, and circle; *volume* of a rectangular box or circular cylinder, *perimeter* of a polygon, and *circumference* of a circle.

Trigonometry *(2 problems)*
You should be able to demonstrate the following skills:
- Use the Pythagorean Theorem to find the length of the third side of a right triangle, given the lengths of any two sides.
- Use the two standard triangles ($30^o – 60^o – 90^o$ and $45^o – 45^o – 90^o$) to find the values of the six trigonometric relationships for the angles in the two standard triangles.
  - You should know the six trigonometric relationships: sine, cosine, tangent, cotangent, secant, and cosecant.
  - Given a right triangle with lengths of any two of the sides indicated, you should be able to find the sine, cosine, or tangent of each of the angles.
  - Given the sine, cosine, or tangent of an angle, you should be able to sketch a triangle and calculate the other trigonometric values of that angle.
- Convert between degrees and radians.
  - In particular, you should know the radian measures for angles of 0^o, 30^o, 45^o, 60^o, 90^o, 180^o, 270^o, and 360^o.
- Use the unit circle to find the sine, cosine, or tangent of angles in the second, third, and fourth quadrants that are related to the angles in the two standard triangles.
  - In particular, you should be able to find the sine, cosine or tangent of 0^o, 30^o, 45^o, 60^o, 90^o, 120^0, 135^o, 150^o, 180^o, …, 270^o, …, and 360^o.

Symbolic computation of the Derivative *(4 problems – see Sections 2.3 and 2.4 of our text)*

- You should be able to take the derivative of the following kinds of functions, where *a*, *b*, and *c* denote constants:
  - Constant functions: $f(x) = \pi$, $f(x) = 5$, (or $f(x) =$ any other constant)
  - Linear functions: $f(x) = a\,x + b$ (any linear function)
  - $f(x) = ax^2 + bx + c$ (or any polynomial)
  - Power functions: $f(x) = a\,x^n$ (any power function)
    - The power could be negative, such as $f(x) = a/x = ax^{-1}$
  - Constant multiple rule: $f(x) = c \cdot g(x)$
    - For example: $f(x) = c \cdot (ax^2 + bx + c)$
  - $f(x) = b^x$ (an exponential function)
  - $f(x) = e^x$ (a special exponential function)
  - Sum rule: functions formed by taking the sum of simpler functions
    For example: $f(x) = 2^x + x^2 + ln(2$

Sample Questions for this Benchmark

Be sure to review the sample questions from the Study Guide for the Quiz you took during the first week of class. In addition to those questions, the following questions are taken from calculus benchmarks that I've given in past years.

1. Write the expression $\sqrt{x^3}$ in the form x^n.
2. Write an expression for the cube root of x^5 in the form x^n.
3. Write the expression $\frac{1}{x^3}$ in the form x^n.
4. Write the expression $\frac{x^4}{x^7}$ in the form x^n.

   What do fractional exponents mean? What do negative exponents mean?

5. If $g(x) = 3x^2$, find an expression for $g(-7)$.
6. If $g(x) = 3x^2$, find an expression for $g(x+5)$.
7. If $g(x) = 3x^2$, find an expression for $g(\pi)$.
8. If $g(x) = 3x^2$, find an expression for $g(\cos(5))$.
9. If $g(x) = 3x^2$, find an expression for $g(\cos(\pi/2))$.

   Although you are not expected to evaluate cos(5) by hand, you should be able to evaluate the cosine or sine of 0, π/6, π/4, π/3, π/2, π, 2π without using a calculator.

10. Find the slope of the line which passes through the point (3, 5) and goes through the origin.
11. Find the equation of the line which passes through the point (3, 5) and goes through the origin.
12. Find the length of the line segment from the point (3, 5) to the origin.
13. Find the midpoint of the line segment from the point (3, 5) to the origin.

14. Find the slope of the line which passes through the points (3, 5) and (-3, 8).
15. Find the equation of the line which passes through the points (3, 5) and (-3, 8).
16. Find the length of the line segment from (3, 5) to (-3, 8).
17. Find the midpoint of the line segment from (3, 5) to (-3, 8).

18. Find the slope of the line which passes through the points (3, 5) and (3, -2).
19. Find the equation of the line which passes through the points (3, 5) and (3, -2).
20. Find the length of the line segment from (3, 5) to (3, -2).
21. Find the midpoint of the line segment from (3, 5) to (3, -2).

22. Does the line from (4, 5) to (2, 8) pass through the origin? How do you know?
23. Does the line from (3, 5) to (-6, -10) pass through the origin? How do you know?

24. Find the equation of a line which passes through the point (3, 5), and is perpendicular to the line y = -x.

25. Is the line y = 5x + 4 [parallel to, perpendicular to, neither parallel nor perpendicular to] the line x + 5y = 20? Explain.
26. Is the line y = 5x + 4 [parallel to, perpendicular to, neither parallel nor perpendicular to] the line 5y – 10x = 20? How do you know?

27. Find an angle whose tangent is -1.
28. If sin(t) = a, find an expression for sec(t).
29. If sin(t) = a, find an expression for cos(t) in terms of a.

30. Evaluate (find the "exact value of") $\sin(\pi/2)$.
31. Evaluate $\cos(\pi/3)$.
32. Evaluate $\tan(\pi/4)$.
33. Evaluate $\cos(\pi)$.

34. If an equilateral triangle has sides of length 3 units, what are the lengths of its altitude and its base?
35. If an equilateral triangle has sides of length 3 units, what is its area?

36. How many *feet* of fencing are needed to enclose a field which is 76 yards wide and 50 yards long?
37. A rectangular box has a volume of 600 cubic inches, and is 8 inches high. If the length is three times the width, what are the dimensions [length, width, height] of the box?
38. A rectangular box has a volume of 600 cubic inches, and is 8 inches high. If the length is three times the width, what is the surface area of the box?

39. What is the volume of a cylinder if its diameter and its height are both 3 inches?
40. What is the total surface area of a cylinder (including its top, bottom, and curved side) if its diameter and its height are both 3 inches?
41. Take the natural log of both sides of the equation $y = 5e^{3x}$, and simplify the resulting expression.
42. Use logs to transform the equation $y = f(x) = 5x^3$ to a linear equation. What is the slope of the resulting equation?
43. Use logs to transform the equation $y = f(x) = 7x^n$ to a linear equation. Is the resulting equation a semi-log equation or a log-log equation?
44. Consider the equation $y = f(x) = 3e^{4x}$. Transform this equation by taking the natural log on both sides of the equation. What is the slope of the transformed equation?

45. Find the derivative of the function $y = f(x) = -3x + 5$
46. Find the derivative of the function $y = f(x) = 5x^2 - 2x + 7$
47. Find the derivative of the function $y = f(x) = 3x^n$
48. Find the derivative of the function $y = f(x) = -4 / x$
49. Find the derivative of the function $y = f(x) = 15$
50. Find the derivative of the function $y = f(x) = \pi$

51. Find the derivative of the function $y = f(x) = 2^x$
52. Find the derivative of the function $y = f(x) = e^x$

53. Find the derivative of the function $y = f(x) = -3/x + 5x^4$
54. Find the derivative of the function $y = f(x) = 4\ (5x^2 - 2\ x + 7)$
55. Find the derivative of the function $y = f(x) = 3\ x^n + 2^x - 1/x^3$
56. Find the derivative of the function $y = f(x) = 4 / x^3$
57. Find the derivative of the function $y = f(x) = ln(5)$
58. Find the derivative of the function $y = f(x) = \pi / (4x)$
59. Find the derivative of the function $y = f(x) = 2^x - 3/x^2 + 5x^4$
60. Find the derivative of the function $y = f(x) = 15\ e^x$

61. Find the derivative of the function $y = f(x) = -3\ x + 5 + sqrt(x) - 15/(2x)$
62. Find the derivative of the function $y = f(x) = 5x^2 - 2\ x + 7$
63. Find the derivative of the function $y = f(x) = 3x^{15}$
64. Find the derivative of the function $y = f(x) = -4$
65. Find the derivative of the function $y = f(x) = 15\ (\pi^x)$
66. Find the derivative of the function $y = f(x) = \pi\ x^3 + 4x$
67. Find the derivative of the function $y = f(x) = 3^x + x^3$
68. Find the derivative of the function $y = f(x) = e^x + 7x - 12/x$

Answer Key for Benchmark Study Questions

1. $x^{3/2}$
2. $x^{5/3}$
3. x^{-3}
4. x^{-3}

   Fractional exponents denote reciprocals; negative exponents denote roots.

5. 147
6. $3(x+5)^2$
7. $3\pi^2$
8. $3\cos^2(5)$
9. $3\cos^2(\pi/2) = 3 \cdot 0^2 = 0$

   Although you are not expected to evaluate cos(5) by hand, you should be able to evaluate the cosine or sine of 0, π/6, π/4, π/3, π/2, π, 2π without using a calculator.

10. 5/3
11. $y = \frac{5}{3}x$
12. $\sqrt{3^2+5^2} = \sqrt{34}$
13. $(3/2, 5/2) = (1.5, 2.5)$
14. $-3/6 = -1/2$
15. $y = -(1/2)x + (13/2) = -0.5x + 6.5$
16. $\sqrt{6^2+3^2} = \sqrt{45} = 3\sqrt{5}$
17. (0, 6.5)
18. The slope is undefined because this is a vertical line. In other words, the "run" is zero.
19. $x = 3$
20. $\sqrt{0^2+7^2} = 7$
21. *(3, 1.5)*
22. The slope of the line from (4, 5) to (2, 8) is -3/2, and the slope of the line from (4, 5) to the origin is 5/4; so we know that the line between (4, 5) and (2, 8) does not pass through the origin.
23. The slope of the line from (3, 5) to (-6, -10) is 5/3, and the slope of the line from (3, 5) to the origin is 5/3; so we know that the line between (3, 5) and (-6. -10) does pass through the origin.
24. $y = x + 2$
25. The slope of y = 5x + 4 is 5, and the slope of x + 5y = 20 is -1/5, so we know that these two lines are perpendicular. The product of their slopes is -1.

26. The slope of y = 5x + 4 is 5, and the slope of 5y – 10x = 20 is 2, so we know that these two lines are neither parallel nor perpendicular.
27. The angle must be in either the second or fourth quadrant; possible answers are $-\pi/4$ and $3\pi/4$. Other answers are possible.
28. $\frac{1}{\sqrt{1-a^2}}$
29. $\sqrt{1-a^2}$

30. 1
31. ½ = 0.5
32. 1
33. -1
34. $altitude = \frac{3\sqrt{3}}{2}, base = 3$
35. Area = $\frac{9\sqrt{3}}{4}$
36. *756 feet*
37. Length = 15 inches, width = 5 inches, height = 8 inches

38. *2 (40 + 75 + 120) square inches*
39. $V = \frac{27\pi}{4}$ in^3
40. $SA = \frac{9\pi}{2} + 9\pi$
41. ln(*y) = ln(5) + 3x*.
42. ln(*y) = ln(5) + 3 ln (x).* The transformed equation is linear, and has slope of 3.
43. ln(*y) = ln(7) + n ln (x).* This is a log-log equation.
44. ln(*y) = ln(3) + 4x*. The slope of this transformed equation is 4.

45. *f'(x) = dy/dx = –3*
46. *f'(x) = dy/dx = 10 x - 2*
47. *f'(x) = dy/dx = 3n* x^{n-1}
48. *f'(x) = dy/dx = 4 /* x^2
49. *f'(x) = dy/dx = 0*
50. *f'(x) = dy/dx = 0*
51. *f'(x) = dy/dx = ln(2)* 2^x
52. *f'(x) = dy/dx =* e^x

53. *f'(x) = dy/dx = 3 /* x^2 *+ 20*x^3
54. *f'(x) = dy/dx = 4 (10x–2) = 40x – 8*
55. *f'(x) = dy/dx =3n* x^{n-1} + ln(2) 2^x *+ 3/*x^4
56. *f'(x) = dy/dx = -12 /* x^4 *= -12* x^{-4}

57. $f'(x) = dy/dx = 0$ (since $ln(5)$ is a constant)
58. $f'(x) = dy/dx = - \pi / (4x^2) = (-\pi) / (4x^2)$
59. $f'(x) = dy/dx = ln(2)2^x + 6 / x^3 + 20 x^3$
60. $f'(x) = dy/dx = 15 e^x$

61. $f'(x) = dy/dx = -3 + 0 + 1 / (2 sqrt(x)) + 15/(2x^2)$
62. $f'(x) = dy/dx = 10 x - 2 + 0 = 10 x - 2$
63. $f'(x) = dy/dx = 45 x^{14}$
64. $f'(x) = dy/dx = 0$
65. $f'(x) = dy/dx = 15 ln(\pi) (\pi^x)$
66. $f'(x) = dy/dx = 3 \pi x^2 + 4$
67. $f'(x) = dy/dx = ln(3) 3^x + 3 x^2$
68. $f'(x) = dy/dx = e^x + 7 + 12 / x^2$